

Backaction driven transport of Bloch oscillating atoms in ring cavities

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We predict that an atomic Bose-Einstein condensate strongly coupled to an intracavity optical lattice can undergo resonant tunneling and directed transport when a constant and uniform force is applied. The bias force induces Bloch oscillations, causing amplitude and phase modulation of the lattice which resonantly modifies the site-to-site tunneling. For the right choice of parameters a net atomic current is generated. The direction and amplitude of the transport velocity depend on the detuning between the pump laser and the cavity, and transport can be enhanced through imbalanced pumping of the two counter-propagating running wave cavity modes. Our results add to the cold atoms quantum simulation toolbox, with implications for quantum sensing and metrology.

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Periodic potentials play a prominent role in condensed matter systems, and highlight some of the fundamental differences between classical and quantum dynamics: a quantum particle undergoes strong scattering when its de Broglie wavelength satisfies the lattice Bragg condition, and can undergo tunneling through classically forbidden regions between sites. Furthermore, if a constant bias force F is applied a quantum particle is not transported in the direction of the force but instead performs Bloch oscillations with no net displacement at a frequency $\omega_B = Fd/\hbar$, where d is the lattice period [1]. Indeed, transport of electrons in lattices with an applied DC electric field only occurs as a result of dephasing processes such as scattering from lattice defects.

Cold atoms present an especially attractive platform for studies of lattice systems because all of the critical parameters governing the dynamics are tunable in real time. In particular, it is possible to control tunneling and transport by modulating the potential in time. Transport in statistical phase space has been demonstrated in pulsed lattices, realizing the quantum delta-kicked rotor [2] and leading to dynamical localization [3] and chaos-assisted tunneling [4]. Directed transport has been observed through ratchet effects in driven dissipative [6] and Hamiltonian lattices [7]. Tunneling control has been achieved through harmonic shaking of lattices without [8–10] and with [11, 12] a bias force. It is thus possible to control the superfluid–Mott insulator transition [13, 14] and to induce macroscopic delocalization [15] and transport [16] of Bloch oscillating atoms. Recently photon-assisted tunneling [17] has been studied in strongly correlated quantum gases [18, 19], and artificial vector gauge potentials have been generated [20].

What is missing in the above schemes is backaction by the atoms upon the electromagnetic fields generating the lattice. Well known backaction effects by electrons in condensed matter systems include lattice-phonon

mediated Cooper pairing and the Meissner effect in superconductors. However, backaction in cold atom systems can be achieved by generating the optical lattice in a high-finesse cavity, which can greatly increase the atom-photon coupling. In the regime of collective strong coupling, the intracavity lattice responds dynamically to the atomic density. This backaction on the lattice leads to richer dynamics than is otherwise possible [21–23], such as bistability-driven dynamics in the single-photon regime [24, 25]. In this sense the system can be considered an application of cavity optomechanics [26–28], where collective excitations of the atoms play the role of the material oscillator, which is dispersively coupled to one or more cavity modes [29].

It has been shown theoretically that such systems allow continuous, non-destructive measurements of atomic Bloch oscillations, through detection of intensity and phase modulation of the transmitted light [30, 31]. Despite the backaction on the lattice effected by the atoms, Bloch’s acceleration theorem remains valid, and this modulation of the lattice potential occurs predominantly at the expected Bloch oscillation frequency (*i.e.*, calculated for an equivalent static tilted lattice) and its harmonics. It is therefore natural to ask whether this dynamical modulation of the lattice can drive the renormalization of atomic tunneling which is now familiar from experiments with free-space lattices. The central result of this Letter is to show that interaction driven modulation of an intracavity lattice can lead to directed atomic current with tunable magnitude and direction.

To see how this happens, we consider a Bose-Einstein condensate trapped along one leg of an optical ring cavity, as shown schematically in Fig. 1. Transverse degrees of freedom are assumed to be frozen out by external confinement, effectively reducing the dynamics to a single spatial dimension z . The two running-wave modes of the cavity are pumped through a lossless input-output cou-

pling mirror by a laser with frequency $\omega_0 = ck_r$, where c is the speed of light in vacuum and $\hbar k_r$ is the recoil momentum. The light is detuned far enough from the atomic resonance that the excited state of the atoms can be adiabatically eliminated. For simplicity we also ignore atomic collisions, which may be negligible in an experiment either because the scattering cross-section is naturally small [15], or has been made small through the use of a tunable Feshbach resonance [16]. In a frame rotating at ω_0 , and in the dipole and rotating wave approximations, the Hamiltonian is then given by

$$\hat{H} = -\hbar \sum_{k=\pm} \left[\Delta_c \hat{a}_k^\dagger \hat{a}_k + i(\eta_k^* \hat{a}_k - \eta_k \hat{a}_k^\dagger) \right] + \int dz \hat{\psi}^\dagger \left(-\frac{\hbar^2}{2m} \partial_z^2 + \hbar U_0 \hat{\mathcal{E}}^\dagger \hat{\mathcal{E}} - Fz \right) \hat{\psi} \quad (1)$$

where the annihilation operators \hat{a}_+ and \hat{a}_- acting on the cavity modes, and $\hat{\psi}(z)$ acting on the atomic field, all obey bosonic commutation relations. $\Delta_c = \omega_0 - \omega_c$ is the detuning of the driving laser from the bare cavity resonance frequency ω_c , and $\eta_k = \sqrt{J_k \kappa}$ characterizes the pump rate for a constant incident flux of J_k photons per unit time and a photon number decay rate of $2\kappa \langle \hat{a}_k^\dagger \hat{a}_k \rangle$ for each mode. The dimensionless positive-frequency component of the electric field is given by $\hat{\mathcal{E}}(z, t) = \exp(-i\omega_c t) [\hat{a}_+ \exp(ik_r z) + \hat{a}_- \exp(-ik_r z)]$. The depth of the lattice is proportional to U_0 , which is a function of the atomic dipole moment, the cavity mode volume and the detuning from atomic resonance, and $F < 0$ is the uniform and constant force inducing the Bloch oscillations.

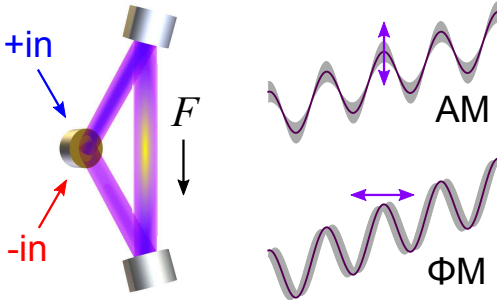


FIG. 1: An optical lattice is created by pumping the two running wave modes of a ring cavity. A trapped Bose-Einstein condensate (yellow) undergoes Bloch oscillations due to the bias force F . Atomic backaction leads to lattice amplitude and phase modulation (AM and ΦM , respectively), which in turn induces coherent directed transport of the condensate.

We can alternatively express the cavity modes in the standing wave basis, described by the annihilation operators $\hat{a}_c = (\hat{a}_+ + \hat{a}_-)/\sqrt{2}$ and $\hat{a}_s = i(\hat{a}_+ - \hat{a}_-)/\sqrt{2}$, where the c (s) mode has cosine (sine) spatial symmetry. In this basis the interaction term in the second line of (1) takes the form

$$\hat{H}_i = \hbar U_0 \left[\hat{n} \hat{N} + (\hat{n}_c - \hat{n}_s) \mathcal{C} + (\hat{a}_c^\dagger \hat{a}_s + \hat{a}_s^\dagger \hat{a}_c) \mathcal{S} \right] \quad (2)$$

where $\hat{n} = \hat{n}_c + \hat{n}_s = \hat{a}_c^\dagger \hat{a}_c + \hat{a}_s^\dagger \hat{a}_s$ is the total number of photons, \hat{N} is the number of atoms in the condensate, and $\mathcal{C}[\hat{\psi}] = \langle \cos(2k_r z) \rangle$ and $\mathcal{S}[\hat{\psi}] = \langle \sin(2k_r z) \rangle$ depend implicitly on the atomic state. We will be most interested in cases where $\langle \hat{n}_c \rangle \gg \langle \hat{n}_s \rangle$, so that the intracavity lattice has predominantly cosine symmetry. Then the quantity \mathcal{C} characterizes the degree of spatial ordering of the atoms and \mathcal{S} is related to the coherence between the lowest and first excited Bloch bands [30]. Viewed in the optomechanical picture \mathcal{C} and \mathcal{S} represent the occupations of the lowest momentum modes of the condensate, with $\mathcal{S} = 0$ in the absence of Bloch oscillations or a symmetry-breaking optical bistability [32]. However even if we choose the η_k to initially give $\mathcal{S} = 0$, during Bloch oscillations \mathcal{S} becomes nonzero and the intensity and spatial phase of the lattice vary dynamically.

To solve the full nonlinear dynamics we write the Heisenberg-Langevin equations, $i\hbar \partial_t \hat{a}_\mu = [\hat{a}_\mu, \hat{H}] - i\hbar \kappa \hat{a}_\mu$ for $\mu = c, s$ and $i\hbar \partial_t \hat{\psi} = [\hat{\psi}, \hat{H}]$, ignoring all input noise operators, whose means are zero, and neglecting atom losses over the time scales of interest so that $N = \langle \hat{N} \rangle$ is constant. Letting $\alpha_\mu = \langle \hat{a}_\mu \rangle$ and $\psi = \langle \hat{\psi} \rangle / \sqrt{N}$, and factoring the expectation values of operator products, we obtain the mean-field equations,

$$\partial_t \alpha_c = -(\kappa - i\Delta_+) \alpha_c - iU_0 N \mathcal{S} \alpha_s + \eta_c \quad (3)$$

$$\partial_t \alpha_s = -(\kappa - i\Delta_-) \alpha_s - iU_0 N \mathcal{S} \alpha_c + \eta_s \quad (4)$$

$$i\hbar \partial_t \psi = \left(-\frac{\hbar^2}{2m} \partial_z^2 + \hbar U_0 |\mathcal{E}(z)|^2 - Fz \right) \psi \quad (5)$$

Here $\Delta_\pm = (\Delta_c - U_0 N) \mp U_0 N \mathcal{C}$ are the effective detunings, and $\mathcal{E}(z, t) = \langle \hat{\mathcal{E}}(z, t) \rangle$ is the dimensionless electric field. The standing-wave modes are pumped at rates $\eta_c = (\eta_+ + \eta_-)/\sqrt{2}$ and $\eta_s = i(\eta_+ - \eta_-)/\sqrt{2}$. Again we see that the lattice modulation is driven by changes in \mathcal{C} and \mathcal{S} during Bloch oscillations — \mathcal{C} drives amplitude modulation through changes of the Stark detuning Δ_+ of the dominant cosine mode, and \mathcal{S} induces shaking of the lattice (*i.e.*, phase modulation) through coherent coupling of the sine and cosine modes.

In Fig. 2, we show the dynamics for ^{88}Sr atoms accelerating under gravity in a 689 nm ring cavity lattice (*i.e.*, lattice spacing $d = \pi/k_r = 344.5$ nm). In this case we have $\omega_B = 2\pi \times 745$ Hz, and the recoil frequency $\omega_r \equiv \hbar k_r^2 / (2m) = 2\pi \times 4.78$ kHz. Here we choose $U_0 N = -\kappa$ corresponding to collective strong coupling, and compare the dynamics with $\kappa = 2\pi \times 1$ kHz and 1 MHz. Bloch oscillations induce lattice amplitude modulation of $\sim 10\%$ peak-to-peak in both cases, with negligible shaking. The fast oscillations on top of the main modulation observed for larger κ were identified previously, and are predominantly higher harmonics of ω_B associated with the nonlinearity of the coupled atom-light system [31]. In the case of small κ these features are outside the cavity bandwidth and therefore suppressed.

Despite similar modulation depths for small and large κ in Fig. 2, transport is only observed when κ is on the order of ω_B . The difference between the two cases is in the

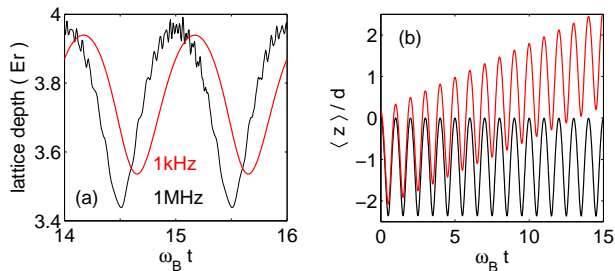


FIG. 2: Backaction induced lattice modulation and atomic transport. Only the cosine cavity mode was pumped, with $U_0 = -2\pi \times 1$ Hz and $\Delta_c = U_0 N = -\kappa$. (a) Lattice depth in units of the recoil energy $E_r = \hbar\omega_r$ as a function of time during intracavity Bloch oscillations. The black curve is for $\kappa = 2\pi \times 1$ MHz, and the red curve for 1 kHz. (b) Condensate centroid position as a function of time, showing *uphill* transport for $\kappa \sim \omega_B$. Colors are the same as in (a).

relative phase delay between the lattice modulation and the Bloch oscillation. This dependence on phase is well known from work with free-space lattices, but in such cases the phase is controlled directly with the applied modulation. When the modulation arises optomechanically, it is set self-consistently according to Eqs. (3)–(5). We will elaborate on this below.

First it is useful to summarize what is known about Bloch oscillations in a free-space optical lattice under harmonic driving of the lattice amplitude or phase, or of the bias force. For concreteness we consider sinusoidal amplitude modulation at ω_B , but shaking of the lattice or modulation of F give similar results [35–37]. For modulation at ω_B we can restrict ourselves to nearest-neighbor tunneling (see Supplementary Material [38]). The transport velocity is equal to the offset of the group velocity after an integer number of Bloch periods, given by

$$v_t = dT_1 \cos \phi \quad (6)$$

where T_1 is the tunneling rate between neighboring sites, which is proportional to the modulation depth, and ϕ describes the lag between the applied modulation and the Bloch oscillation [39, 40]. For initial site-to-site coherence, such as we consider here, the result is directed transport superposed on the underlying Bloch oscillation [16]. Given initially random site-to-site phases, one instead observes spatial spreading of the atoms [11, 15].

Returning to the intracavity case we recall from Fig. 2 that modulation of the condensate density leads to backaction induced amplitude modulation of the lattice at ω_B . If one imagines the lattice as a damped oscillator driven by the atomic Bloch oscillation, then the condition $\omega_B \sim \kappa$ offers the best balance of amplitude response and relative phase for inducing transport. For these parameters the modulation phases are $\phi \approx -90^\circ$ for large κ and $\phi \approx -150^\circ$ for small κ , leading to the observed transport behavior in a manner consistent with Eq.(6) for free-space lattices. To explore this connection further

we have extended the theory for free-space lattices to include the self-consistent optomechanical dependence of T_1 and ϕ [38]. The analytic theory is in excellent agreement with the numerical simulations, predicting near optimum phase when $\kappa \sim \omega_B$, as well as uphill transport over a range of detunings.

The magnitude and direction of transport depend on the cavity pumping parameters Δ_c and η_{\pm} , as well as the Stark shift $U_0 N$. We find that $|v_t|$ increases quadratically with $U_0 N$ for small values, and is maximized for intermediate lattice depths ($\sim 3\hbar\omega_r$ for the parameters of Fig. 2). In very shallow lattices, tunneling is strong but the modulation depth is reduced in proportion to the trap depth; in deep lattices tunneling is suppressed and the modulation depth is reduced due to flattening of the Bloch bands. The dependence on detuning Δ_c is shown in Fig. 3. For balanced pumping, with $\eta_+ = \eta_-$ ($\eta_s = 0$), the transport exhibits a dispersive shape around the Stark-shifted cavity resonance ($\Delta_+ = 0$). This is because modulation of \mathcal{C} during the Bloch oscillation effectively dithers the detuning of the cosine cavity mode, as described by Eq. (3). The result is a modulation amplitude which approximately follows the derivative of the Lorentzian cavity response. The modulation phase also changes across the resonance, adding to the detailed shape we observe. Again these effects are well captured by the analytic tight-binding theory. We note in passing that for larger blue detunings, Raman transitions of the type studied in [41] can become resonant, leading to Rabi oscillations between condensate momentum states and tunneling into higher bands.

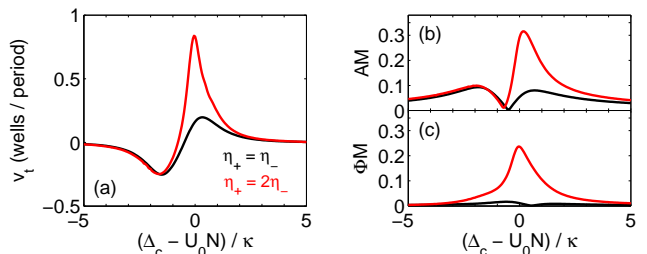


FIG. 3: Control of transport with detuning. (a) Transport velocity as a function of Δ_c , for the same parameters as Fig. 2 and $\kappa = 2\pi \times 1$ kHz. The black curve is for balanced pumping, and the red curve for a strong imbalance. (b) Relative depth of self-induced lattice amplitude modulation (AM), defined as half the peak-to-peak lattice depth modulation, normalized to the mean lattice depth. (c) Phase modulation (Φ_M) defined as half the peak-to-peak spatial extent of the lattice shaking, in units of the lattice spacing.

Because the effects we have described so far are dominated by lattice amplitude modulation, they are qualitatively present in standing wave cavities as well. However, only in ring cavities is it possible to pump the counter-propagating running-wave modes with independent amplitudes, corresponding to direct pumping of the sine mode (not to be confused with a translation of the lattice,

such as occurs during lattice shaking). In Fig. 3(a) we observe a strong enhancement of uphill transport around $\Delta_c = U_0N$ when $\eta_+ = 2\eta_-$, even though the initial trap depth is the same as before. As seen in Figs. 3(b) and (c), both the amplitude and phase modulations are increased. Imbalanced pumping increases population of the cavity sine mode, thereby enhancing the backaction induced lattice shaking. At first it may be surprising that this effect is only pronounced around $\Delta_c = U_0N$, but the effective detuning of the cosine mode Δ_+ becomes positive here, corresponding to the regime of cavity heating [42], where linear stability analysis for $F = 0$ predicts unstable dynamics [43]. In contrast, near the positive transport peak one finds that both the sine and cosine modes operate in the cavity cooling regime with Δ_{\pm} negative, where the $F = 0$ dynamics are damped.

The effect of imbalanced pumping is investigated further in Fig. 4, where we vary the ratio η_+/η_- , for $\Delta_c = U_0N$ and fixed initial trap depth. Phase modulation is more sensitive than amplitude modulation to small pump asymmetries. We observe that the transport minima are slightly offset from the condition of balanced pumping; the actual minima occur where the time-averaged value of \mathcal{S} over a full Bloch oscillation period vanishes. This is due to weak pumping of the sine mode balancing the atomic dynamics. Simulations at strong imbalances reveal the existence of numerous types of instability. These include Landau-Zener tunneling, symmetry-breaking bistability [32], and collective excitations of the condensate [43], and will be the subject of a future work [44]. Near the onset of instability, the depths of amplitude and phase modulation as defined here become comparable; recall Figs. 3(b) and (c). However we note that AM is more efficient at driving transport when each type of applied modulation is considered in isolation for free-space lattices. Transport appears to be dominated by amplitude modulation in all of the parameter regimes we have studied, with a four-fold increase in $|v_t|$ possible through imbalanced pumping of the cavity.

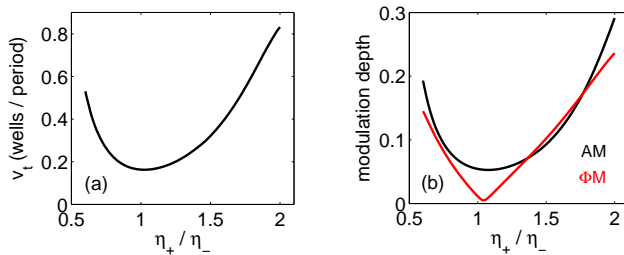


FIG. 4: Effects of imbalanced pumping of the running-wave cavity modes. (a) Transport velocity as a function of η_+/η_- , with $\Delta_c = U_0N$ and the same parameters as Fig. 3. (b) Amplitude and phase modulation depths as defined above.

We can now compare our results to experiments in driven free-space lattices. Combined Bloch oscillations

and transport have been observed by imaging atoms both *in situ* and in time-of-flight [15, 16]. For intracavity lattices one could additionally detect the Bloch oscillations non-destructively in the transmitted light [30, 31], but our calculations suggest that transport would not be readily apparent. A remarkable feature of the backaction driven dynamics is that ω_B appears to remain unchanged from its value in a static lattice. This was previously shown for pure AM in standing-wave cavities [31], but it is significant that it remains true in the presence of ΦM . This is critical for applications in metrology and sensing where the modulation frequency should be directly proportional to the applied force [30]. It also implies that backaction will not induce so-called super Bloch oscillations [16], because these occur when the lattice modulation is detuned from ω_B . It is worth noting that although the backaction driven ΦM is much smaller than what has been applied in free-space experiments [45], the values of AM we observe are similar to what was applied in Refs. [36, 37]. Finally, we note that our tight-binding theory predicts that in the absence of initial site-to-site coherence, the modulation will decay. This is because \mathcal{C} and \mathcal{S} become constant, cutting off the backaction induced modulation of the lattice according to Eq. (2).

In conclusion, we have shown that optomechanical effects lead to qualitatively new dynamics for a Bose-Einstein condensate undergoing Bloch oscillations in a high finesse optical cavity. As the condensate quasimomentum samples the first Brillouin zone, the optical lattice depth and position are dynamically modulated, even as the Bloch frequency itself is unchanged. When the cavity damping rate is on the order of the Bloch oscillation frequency, coherent directed transport of the condensate can be observed. Asymmetric pumping of the running wave cavity modes enhances the transport, and in extreme cases makes the system dynamically unstable. Our results extend the study of coherent control of tunneling to include optomechanical lattice excitations. They are relevant to measurements of Bloch oscillations in cavities and other non-destructive atomic probes, and also more generally to attempts to realize neutral atom quantum simulators where backaction upon electromagnetic fields is significant.

Note added: since the preparation of this manuscript, we have become aware of the experimental observation of transport of a Bose-Einstein condensate in a high-finesse standing wave cavity [46].

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